PARAMETER IDENTIFICATION FOR NONLINEAR AERODYNAMIC SYSTEMS

From

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1. Introduction

This Progress Report covers the six month period from April 24 to October 22, 1991. The completed work during this period is contained in the two papers listed in Section 4 and summarized below. Work continues on frequency analysis for transfer function identification, both with respect to the continued development of the underlying algorithms and in the identification study of two physical systems. In addition, some new results of a theoretical nature have recently been obtained that lend further insight into the frequency domain interpretation of this research. Progress in each of these areas is summarized below. Although not related to the system identification problem, some new results have been obtained on the feedback stabilization of linear time lag systems.

2. List of Scientific Collaborators: April 24 to October 22, 1991:

A. V. Fullerton* Graduate Student Research Assistant
J. Q. Pan Graduate Student Research Assistant
A. A. Pandiscio Raytheon Graduate Student Fellow
A. E. Pearson* Professor and Principal Investigator
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3. Completed and Continuing Research

3.1. Parameter Identification for Exact Differential Systems

A paper has been written entitled "Explicit Least Squares System Parameter Identification for Exact Differential Input/Output Models" which emphasizes the *explicit* nature of the modulating function approach to parameter identification for systems that can be modeled by an *exact* differential operator equation of the generic form:

$$\sum_{j=0}^{n_1} \sum_{k=1}^{n_2} g_j(\theta) P_{jk}(p) E_k(u,y) = 0 \qquad (1)$$

where θ denotes a vector of parameters, $P_{jk}(p)$ is a polynomial in the differential operator $p = d/dt$, $[u(t), y(t)]$ in an input/output (i/o) data pair observed over some time interval $[0,T]$, and (g_j, P_{jk}, E_k) are given functions of their arguments that depend on the specified model. As listed in Section 4, this paper will appear in the Proceedings of the Eighth International Conference on Mathematical and Computer Modelling.[1] The algorithm described therein utilizes the "real-valued" trigonometric modulating functions previously detailed in our earlier progress reports. This is in contrast with some new insight recently obtained through the use of the "complex-valued" modulating functions. For the purpose of describing this insight in some detail, consider the following set of order n modulating functions defined on a time interval $[0,T]$:

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[1] Three preprint copies of this paper were mailed to Dr. P. C. Murphy of the Aircraft Guidance and Control Branch on September 13, 1991.

$$\phi_m^n(t)=e^{-im\,\omega_0 t}(e^{-i\,\omega_0 t}-1)^n, \quad 0\leq t \leq T=2\pi/\omega_0 \tag{2}$$

where $i=\sqrt{-1}$, m is any integer, $\omega_0=2\pi/T$ plays the role of a "resolving frequency", and n corresponds to the order of the differential equation under investigation, e.g., the highest degree of the polynomials $P_{jk}(p)$ appearing in the model (1).[2]

Noting from the binomial expansion that $\phi_m^n(t)$ has the equivalent representation

$$\phi_m^n(t)=e^{-im\,\omega_0 t}\sum_{k=0}^{n}(-1)^{n-k}\binom{n}{k}e^{-ik\,\omega_0 t} \tag{3}$$

it has been shown that the **Modulation Property** for this set of functions has the following (new) characterization:

Let $P(p)$ be a linear differential operator of order n and $z(t)$ any sufficiently smooth function defined on $[0,T]$. Then the integration of $\phi_m^n(t)P(p)z(t)$ over $[0,T]$ satisfies

$$\int_0^T \phi_m^n(t)P(p)z(t)dt=\Delta^n P(im\,\omega_0)Z(m) \tag{4}$$

where $Z(m)$ is the Fourier series coefficient for the mth harmonic of $z(t)$, i.e.,

$$Z(m)=\int_0^T z(t)e^{-im\,\omega_0 t}dt$$

and Δ^n is the nth order finite difference operator, i.e., with $Q(m)=P(im\,\omega_0)Z(m)$,

$$\Delta Q(m)=Q(m+1)-Q(m)$$

$$\Delta^2 Q(m)=Q(m+2)-2Q(m+1)+Q(m)$$

$$\Delta^n Q(m)=\sum_{k=0}^{n}(-1)^k\binom{n}{k}Q(n+m-k).$$

Application of the above property to single input/output linear differential system models leads to the block diagram in Fig. 1. This diagram indicates the flow of calculations that are involved in setting up a standard least squares regression for estimating the parameters $\theta=\text{col}\,[-a_1, \cdots -a_n, b_1, \cdots b_n]$ in the regression model:

$$\gamma_0(m)=\gamma(m)\theta, \quad m=0,1,2\cdots \tag{5}$$

where the row vector $\gamma(m)$ of regressors is defined by

$$\gamma(m)=\text{row}[\gamma_1^y(m), \cdots \gamma_n^y(m), \gamma_1^u(m), \cdots \gamma_n^u(m)] \tag{6}$$

and the $(\gamma_j^u(m), \gamma_j^y(m))$ are defined in the diagram. The main difference between the least

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[2] The fact that each $\phi_m^n(t)$ is an nth order modulating function on $[0,T]$ stems from the property that this function and its first $n-1$ derivatives vanish at both end points, i.e., $p^k\phi_m^n(t)=0$ at $t=0$ and $t=T$, $k=0,1\cdots n-1$.

squares estimation based on Eq. (5) and our previous formulation lies in the role of the frequency index m. Thus, the Fourier series coefficient sequence pair $[U(m),Y(m)]$, which can be efficiently and accurately calculated by DFT/FFT techniques given the i/o data $[u(t),y(t)]$ on $0 \leq t \leq T$, is appropriately displayed as the fundamental data-related quantity upon which the Fourier modulating function technique is based. As shown in the diagram, the nth order finite difference operators and the cascade of frequency modulators which operate on these quantities to produce the regressors $\gamma(m)$ clearly show the tradeoff in going from the differential equation model to the corresponding Fourier series frequency domain equation model. Previously, this "discrete" frequency domain interpretation was masked by the vector-matrix formulation of the real-valued Fourier based modulating functions. Although this interpretation does not change any of the results thus far obtained, in particular the one-shot least squares estimation should be identical, it throws the sequential least squares estimation into the proper light and may lead to new results in characterizing conditions for uniqueness of the estimate. For example, the null space of the nth order finite difference operator is the space of all polynomials $Q(m)$ of degree $n-1$, and this fact immediately implies that the i/o data must contain at least n frequencies else the regressors will not be linearly independent. Another possibility is to model the residuals as a parametrized stochastic sequence, e.g., Poisson, in the discrete frequency domain with an attempt to estimate the parameters for this process in addition to the system parameters.

3.2. Parameter Identification for Inexact Differential Systems

Another interpretation resulting from the above **Modulation Property** is the least squares formulation using the modulating functions (2)-(3) for the class of *inexact* differential operator models defined generically by:

$$\sum_{j=0}^{n_1} \sum_{k=1}^{n_2} g_j(\theta) F_{jk}(u,y) P_{jk}(p) E_k(u,y) = 0. \tag{7}$$

The difference between this more general class of i/o models and the *exact* model (1) is the allowance for a data dependent term $F_{jk}(u(t),y(t))$ multiplying the differential operator polynomials $P_{jk}(p)$. For example, a particle of mass m and displacement $y(t)$, subject to a given force $u(t)$ and drag proportional to velocity squared, can be modeled by

$$m\ddot{y} + \theta_1 (\dot{y})^2 - \theta_2 u = 0.$$

Utilizing the differential identity: $p^2(y^2) = 2yp^2 y + 2(py)^2$, the preceding model can be rewritten into the following equivalent vector-matrix equation which is of the form (7):

$$[m,\theta_1,\theta_2] \begin{bmatrix} p^2 & 0 & 0 \\ -yp^2 & \tfrac{1}{2}p^2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} y \\ y^2 \\ u \end{bmatrix} = 0.$$

Given the i/o data $[u(t),y(t)]$ on $[0,T]$ and the data-related signals $w_{jk}(t) = F_{jk}(u(t),y(t))$ on $[0,T]$, assume that each $w_{jk}(t)$ has the Fourier series representation:

$$w_{jk}(t) = \sum_{l=-\infty}^{l=\infty} W_{jk}(l) e^{il\omega_0 t}.$$

It can be shown that application of the **Modulation Property** (4) to the model (7) leads to the following explicitly defined function for least squares minimization:

$$J(\theta) = \sum_{j,k=0}^{n_1} g_j(\theta) g_k(\theta) R_{jk} \tag{8}$$

where the jk^{th} component of the real symmetric nonnegative definite matrix R is defined by

$$R_{jk} = \sum_{m=0}^{M} \lambda_m \tilde{\gamma}_j(m) \tilde{\gamma}_k^*(m) \tag{9}$$

with specified nonnegative "frequency" weights λ_m. Given the Fourier series coefficients $V_k(m)$ of the data-related functions $v_k(t) = E_k(u(t), y(t))$ on $[0,T]$, i.e.,

$$V_k(m) = \int_0^T v_k(t) e^{-im\omega_0 t} dt,$$

the regressors $\tilde{\gamma}_j(m)$ are obtained by the following equations:

$$\tilde{\gamma}_j(m) = \sum_{k=1}^{n_2} W_{jk}(m) \otimes \gamma_{jk}(m)$$

where

$$\gamma_{jk}(m) = \Delta^n Q_{jk}(m)$$

and

$$Q_{jk}(m) = P_{jk}(im\omega_0) V_k(m).$$

The convolution implied above is the linear convolution between Fourier series type coefficients as defined by

$$W(m) \otimes \gamma(m) = \sum_{l=-\infty}^{\infty} W(l) \gamma(m-l).$$

The flow of calculations involved in the above formulation is shown in Fig. 2 which includes the flow of calculations for the more specialized *exact* model (1). Notice that the least squares problem for the *exact* model does not entail any convolutions in the discrete frequency domain. The presence of these convolutions in the case of *inexact* models is consistent with the well-known general property of Fourier analyses in that multiplication in the time domain entails convolution in the frequency domain, and vice versa. Here the frequency domain is the discrete frequency domain of Fourier series coefficients, and the presence of such convolutions imposes a smoothness condition on the "inexact" data-related signals $w_{jk}(t)$ to the extent that only a finite number of the corresponding Fourier series coefficients $W_{jk}(l)$, $l=0, \pm 1, \pm 2, \cdots \pm L$, can be calculated in practice. This L has to be less than the number N of discrete-time samples of the i/o data, $[u(jh)y(jh)]$, $j=0,1 \cdots N$, $Nh=T$, which are used to calculate the Fourier series coefficients $[V_k(m), W_{jk}(m)]$ by DFT/FFT techniques.

3.3. Comparison With Other Techniques

We are currently attempting to compare the Fourier modulating function technique to other available methods in two problem areas: The identification of coefficients in a linear differential system in which the available method is the Prediction Error Method in MATLAB's Identification Toolbox by L. Ljung, and the high resolution frequency estimation problem in which we have coded up the High Order Yule Walker algorithm for simulation purposes. The results are thus far quite encouraging in favor of the Fourier modulating function technique. These results will be written up when completed.

Another area in which we have made some comparisons is the "frequency analysis problem" of estimating the transfer function $G(i\omega)$ for a linear system given the i/o data $[u(t), y(t)]$ over a sequence of time intervals $[t_j, t_j+T]$, $j=0,1,2 \cdots N$, where the modulating function technique facilitates estimation of $G(i\omega)$ at selected knots $\omega=k\omega_0$, $k=0,1 \cdots M$, with the resolving frequency ω_0 related to the lengths of the time intervals by $\omega_0=2\pi/T$. The short paper entitled "Frequency Analysis Via the Method of Moment Functionals" includes comparative data with the classical cross correlation method and the transfer function obtained by the direct ratio of Fourier transforms of the i/o data.[3] Again, the results of these comparisons are quite encouraging. A full length version of this paper will be prepared at some future time. It may be that it will prove advantageous to incorporate the insight obtained by the newly characterized **Modulation Property** described in Eq. (4) above.

3.4. Physical System Identification

Two physical system identification projects are underway and a third is planned for the future. One project is the verification study of a fourth order Butterworth filter that was designed and built by Yan Shen to low-pass filter the data in a wind tunnel setup at Brown University, and another is the dynamic modeling of room acoustics (also being carried out by Yan Shen) which is part of a microphone array project being carried out at Brown under the direction of Prof. Harvey Silverman. These projects are helping "fine-tune" the algorithms by focusing on the practical problems of order determination and computational considerations for high order system models. The first of these is nearing completion and will be submitted in report form in the not-too-distant future.

The third project will focus on modeling for the F-18 aircraft based on data forwarded to us by E. A. Morelli. We shall start this project after we have completed the Butterworth filter verification study.

4. Publications and Presentations

Pearson, A. E., "Explicit Least Squares System Parameter Identification for Exact Differential Input/Output Models." Regular Paper to appear in the Proceedings of the Eighth ICMCM, Univ. of Maryland, College Park, MD, April 1991.

Pearson, A. E. and J. Q. Pan, "Frequency Analysis Via the Method of Moment Functionals."

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[3] Three preprint copies of this paper were also mailed to P. C. Murphy on September 13, 1991.

Short Paper to appear in the Proceedings of the 30th IEEE CDC, Brighton, UK, Dec. 1991.

October 3 Presentation at the University Grantees Workshop, NASA Langley Research Center, Aircraft Guidance and Control Branch.

October 21 Seminar Presentation "Parameter Identification for Differential System Models Via Fourier Modulating Functions" at Northwestern University, Dept. of Electrical Engineering and Computer Science, Evanston, IL.

October 22 Seminar Presentation "Modeling and Parameter Identification for Differential Systems" at the University of Illinois, Dept. of Mechanical Engineering, Urbana-Champaign, IL.

SISO MODEL:

$$p^n y(t) + \sum_{j=1}^{n} a_j \, p^{n-j} y(t) \; - \; \sum_{j=1}^{n} b_j \, p^{n-j} u(t) = 0$$

$$\int_0^T \phi_m^{\,n}(t) \, [\,\text{above}\,] = \Delta^n (im\omega_0)^n Y(m) + \sum_{j=1}^{n} a_j \Delta^n (im\omega_0)^{n-j} Y(m) \; - \; \sum_{j=1}^{n} b_j \Delta^n (im\omega_0)^{n-j} U(m)$$

$$= \gamma_0(m) + \sum_{j=1}^{n} a_j \, \gamma_j^{\,y}(m) \; - \; \sum_{j=1}^{n} b_j \, \gamma_j^{\,u}(m)$$
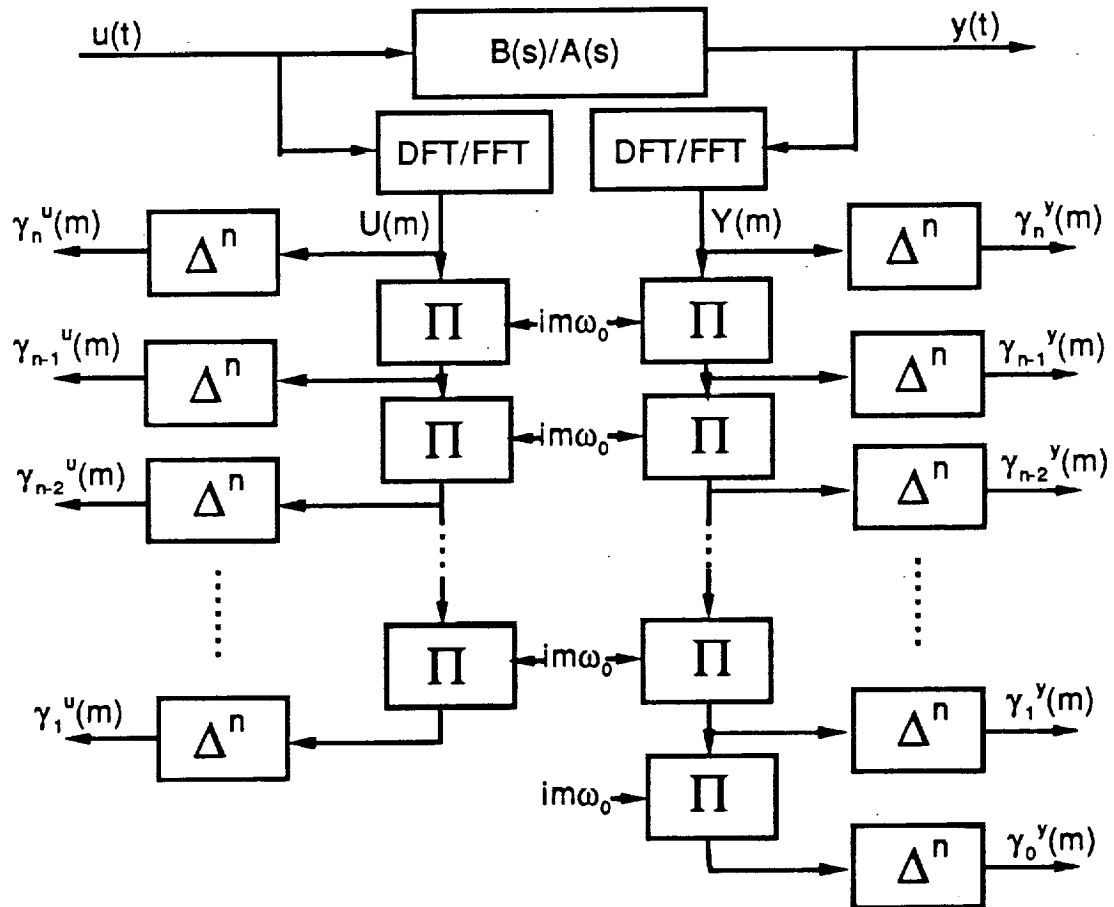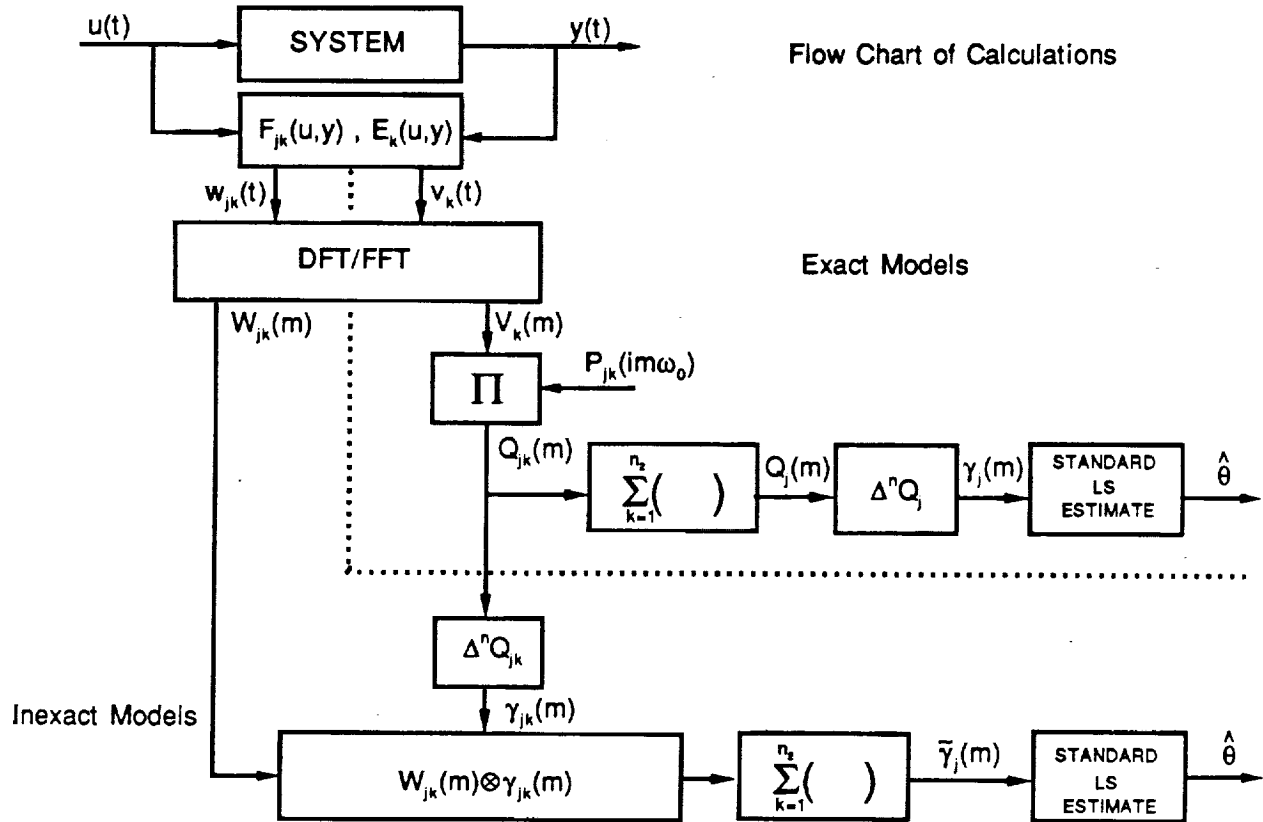

Fig. 1 Linear System Identification

Fig. 2 Nonlinear System Identification